Models Supporting Nondeterminism and Probabilistic Choice

Michael Mislove*

Department of Mathematics
Tulane University
New Orleans, LA 70118
mwm@math.tulane.edu

Abstract. In this paper we describe the problems encountered in building a semantic model that supports both nondeterministic choice and probabilistic choice. Several models exist that support both of these constructs, but none that we know of satisfies all the laws one would like. Using domain-theoretic techniques, we show how a model can be devised "on top of" certain models for probabilistic choice, so that the expected laws for nondeterministic choice and probabilistic choice remain valid.

1 Introduction

Nondeterminism is a standard component of concurrent programming languages. The most widely employed method for modeling concurrent computation takes sequential composition as a primitive operator, from which it is only natural to use nondeterministic choice to generate an interleaving semantics for concurrent computation. This approach also is well-supported by the models of computation that are available. And, there there are some standard assumptions that one expects to hold in any denotational model supporting nondeterministic choice, $+$. For example, a basic assumption one expects to hold is that $+$ is idempotent, commutative and associative, so, in mathematical parlance, we expect $+$ to be a semilattice operation on whatever denotational model we have devised for our language.

If we view internal nondeterminism from a specification point of view, then $p + q$ represents a "don't care" process – we are just as happy to have our program act like p or like q, since either one will (presumably) have the requisite behavior we are seeking. On the other hand, if we are concerned with such things as network flow, then fairness issues arise, and we now might be more concerned with the possibility that one branch dominates, preventing the other from ever having a chance to execute. This is quite different from the view of nondeterminism as under specification, and so some other operation is needed. A notion of probabilistic choice is natural to consider here, and this leads to

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the introduction of probabilistic choice operators. One might think of $p\,_{.5}\!+\,q$ [1] as a probabilistic choice, in which the process acts like p half of the time and like q the other half. Of course, different laws are expected of the operators like $_{.5}\!+$. For example, we do not expect an associative operation – indeed, we expect $(p\,_{.5}\!+\,q)\,_{.5}\!+\,r = p\,_{.25}\!+\,(q\,_{.25}\!+\,r)$.

Interesting aspects emerge when one combines all these operators within a single model. The existing models for process algebras that support both nondeterministic choice and probabilistic choice do not satisfy all the laws that one might expect to hold. Typically, the nondeterministic choice operator is no longer idempotent for all processes. While arguments have been put forth to justify the fact that $+$ is not idempotent on processes which involve probabilistic choice, our view is that idempotence is a property which would be useful to retain.

And this brings us to the issue we are interested in confronting: how to build denotational models for process algebras which have both nondeterministic choice and probabilistic choice, so that the laws for nondeterministic choice and for probabilistic choice that one expects to hold actually are valid.

2 Domains

In this section, we review some of the basics we need to describe our results. A good reference for most of this can be found in [1]. To begin, a *partial order* is a non-empty set endowed with a reflexive, antisymmetric and transitive relation. If P is a partial order, then a subset $D \subseteq P$ is *directed* if every finite subset of D has an upper bound in D. We say P is *directed complete* if every directed subset of D has a least upper bound, $\sqcup D$, in P. Such partial orders we call *dcpos*, and we use the term *cpo* for a dcpo that also has a least element, usually denoted $\bot$.

The functions of interest between dcpos are those that are *Scott continous*. While this can be stated in topological terms, it's just as easy to describe them as those maps that preserve the order and sups of directed sets. The following result is the basis for the "least fixed point semantics" that often is used in domains to model recursion.

Theorem 1 (Tarksi-Knaster-Scott). *If $f\colon P \to P$ is a continuous selfmap of a cpo P, then $\mathrm{FIX}(f) = \sqcup_{n\geq 0} f^n(\bot)$ is the least fixed point of f.* □

The class of (d)cpos and Scott continuous maps is a cartesian closed category.

Definition 1. *Let P and Q be cpos. An embedding-projection pair $(e,p)\colon P \to Q$ is a pair of continuous maps $e\colon P \to Q, p\colon Q \to P$ satisfying $x \leq p(y) \;\Leftrightarrow\; e(x) \leq y$ for all $x \in P$ and $y \in Q$ and $p \circ e = 1_P$.*

[1] We will use the notation $p\,_{\lambda}\!+\,q$ to denote a probabilistic choice in which the process has probability λ of acting like p, and probability $1-\lambda$ of acting like q, where $0 \leq \lambda \leq 1$.

3 CSP

One of the most widely studied process algebras for concurrent computation is CSP, a process algebra which represents processes by the actions in which they can participate. A representative portion of its BNF is given by:

$$p ::= STOP \mid SKIP \mid a \to p \mid p;p \mid p \setminus A \mid p \Box p \mid p \sqcap p \mid p_A\|_B p \mid p|||p \mid x \mid \mu x.p$$

Briefly, $STOP$ is the deadlocked process incapable of any actions, $SKIP$ denotes the process whose only action is normal termination, $a \to p$ is a process which first is willing to participate in the action $a \in \Sigma$, the set of possible actions, and then will act like process p, $p;q$ denotes sequential composition, $p \setminus A$ is the process p with the actions $A \subseteq \Sigma$ hidden from the environment, $p \Box q$ is the process which offers the environment the choice of acting like p or like q, the choice being resolved on the first step, $p \sqcap q$ is the process which internally decides to act like p or like q with no influence from the environment, $p\,{}_A\|_B\, q$ is the parallel composition of p and q synchronizing on all actions in $A \cap B$, and with each branch executing any other actions that it is capable of independently, $p|||q$ is the interleaved parallel composition of p and q, x is a process variable, the set of all such being V, and $\mu x.p$ denotes recursion.

A *finite trace* of a process p is a sequence of actions $a_1 a_2 \cdots a_n$ that p can participate in. An *infinite trace* is an infinite sequence of such actions. It is well known that traces are insufficient to distinguish external and internal choice. This shortcoming was overcome for CSP by the introduction of failures [3].

A *failure* of a process p is a pair $\langle t, X \rangle$, where t is a finite trace of p, and X is a set of actions that p might refuse to participate in, once t is completed.

CSP supports the notion of an internal action, τ, that is used to denote a process p executing an action internally and becoming another process. Since CSP also supports hiding of actions - taking visible actions and making them invisible to the environment, the possibility of *divergence* arises: for example, the process $p = (\mu x.a \to x) \setminus \{a\}$ which becomes eternally engaged in internal actions and never responds to the environment. A *divergence* of a process p is a trace t after which p could become divergent.

The *failures-divergences model* $\mathbb{FD}$ for CSP models each process p as a pair (F, D), where F is the set of failures of p, and D the set of divergences. In order to have a well-behaved denotational model for the set of CSP processes, several healthiness conditions are made on these pairs. The failures set F satisfies the conditions that it is non-empty, if (t, X) is a failure of p, then so is $(s, \emptyset)$ for any prefix s of t, and "impossible events can be added to refusals": if (t, X) is a failure of p and $t\hat{\ }a$ is not a trace of p, then $(t, X \cup \{a\})$ is a failure.

The main criterion for divergences is that once a process has diverged on a trace s, then it can never recover. So, if $s \in D$, then $st \in D$ for all traces t, and (st, X) is a failure of p for every trace t and every subset X of actions. The failures-divergences model supports interpretations of the all the standard operators of CSP.

In $\mathbb{FD}$, the *order of nondeterminism* is used, in which the pair $(F, D) \sqsubseteq (F', D')$ if and only if $F \supseteq F'$ and $D \supseteq D'$. Thus, the higher a pair is in this order, the more deterministic is its behavior. In this model, the deterministic processes form the set of maximal elements (cf. [3]).

4 The probabilistic power domain

The construction that allows probabilistic choice operators to be added to a domain is the probabilistic power domain, which was first investigated by Saheb-Djarhomi [12], who showed that the family he defined yields a cpo. It was refined and expanded by Jones [6, 7] who also was showed that the probabilistic power domain of a continuous domain is again continuous.

Definition 2. *If P is a dcpo, then a* continuous valuation *on P is a mapping $\mu\colon \Sigma D \to [0,1]$ defined on the Scott open subsets of P that satisfies:*

1. $\mu(\emptyset) = 0$.
2. $\mu(U \cup V) = \mu(U) + \mu(V) + \mu(U \cap V)$,
3. *μ is monotone, and*
4. *$\mu(\cup_i U_i) = \sup_i \mu(U_i)$, if $\{U_i \mid i \in I\}$ is an increasing family of Scott open sets.*

We order this family pointwise: $\mu \leq \nu \;\Leftrightarrow\; \mu(U) \leq \nu(U)$ $(\forall U \in \Sigma P)$. We denote the family of continuous valuations on P by $\mathcal{P}_{Pr}(P)$.

Among the continuous valuations on a dcpo, the *simple valuations* are particularly easy to describe. They are of the form $\mu = \Sigma_{x\in F}\, r_x \cdot \delta_x$, where $F \subseteq P$ is a finite subset, δ_x represents point mass at x (the mapping sending an open set to 1 precisely if it contains x, and to 0 otherwise), and $r_x \in [0,1]$ satisfy $\Sigma_{x\in F}\, r_x \leq 1$. In this case, the *support* of μ is just the family F.

A basic result of [6] is that the probabilistic power domain of a continuous domain is continuous. The probabilistic power domain extends to an endofunctor on continuous domains, so each continuous map $f\colon P \to Q$ between (continuous) domains can be lifted to a continuous maps $\mathcal{P}_{Pr}(f)\colon \mathcal{P}_{Pr}(P) \to \mathcal{P}_{Pr}(Q)$ by $\mathcal{P}_{Pr}(f)(\mu)(U) = \mu(f^{-1}(U))$. In fact, [6] shows that the resulting functor is a left adjoint, which means that $\mathcal{P}_{Pr}(P)$ is a free object over P in an appropriate category. The category in question can be described in terms of probabilistic choice operators satisfying certain laws (cf. [6]):

Definition 3. *A* probabilistic algebra *is a dcpo A endowed with a family of Scott continuous operators ${}_\lambda{+}\colon A \times A \to A$, $0 \leq \lambda \leq 1$ such that $(\lambda, a, b) \mapsto a \,{}_\lambda{+}\, b\colon [0,1] \times A \times A \to A$ is continuous and so that the following laws hold for all $a, b, c \in A$:*

- $a \,{}_\lambda{+}\, b = b \,{}_{1-\lambda}{+}\, a$,
- $(a \,{}_\lambda{+}\, b) \,{}_\rho{+}\, c = a \,{}_{\lambda\rho}{+}\, (b \,{}_{\frac{\rho(1-\lambda)}{1-\lambda\rho}}{+}\, c)$ (if $\lambda\rho < 1$).
- *$a \,{}_\lambda{+}\, a = a$, and*
- $a \,{}_1{+}\, b = a$.

The operations ${}_\lambda{+}$ are defined on $\mathcal{P}_{Pr}(P)$ in a pointwise fashion, so for instance, $\mu \,{}_\lambda{+}\, \nu = \lambda\mu + (1-\lambda)\nu$. It then is routine to verify that $\mathcal{P}_{Pr}(P)$ is a probabilistic algebra over P for each dcpo P.

4.1 Probabilistic CSP

The syntax of probabilistic CSP investigated in [9] is not much different from that of CSP – PCSP simply adds the family of operators ${}_{\lambda}+$ for $0 \leq \lambda \leq 1$. So, we now can reason about processes such as $(a \to Stop)\,{}_{.5}\!+ (b \to STOP \,\Box\, c \to STOP)$, which will act like $a \to STOP$ half of the time, and offer the external choice of doing a b or a c the other half.

The model for PCSP that was devised in [9] is built simply by applying the probabilistic power domain operator to the failures-divergences model for CSP. The interpretation of the operators of CSP in $\mathcal{P}_{Pr}(\mathbb{FD})$ is accomplished by analyzing the construction itself. Namely, $\mathcal{P}_{Pr}(\mathbb{FD})$ is a set of continuous mappings from the set of Scott open sets of $\mathbb{FD}$ to the unit interval. So, for example, given a unary operator $f\colon \mathbb{FD} \to \mathbb{FD}$, we can extend this to $\mathcal{P}_{Pr}(\mathbb{FD})$ by $\mathcal{P}_{Pr}(f)\colon \mathcal{P}_{Pr}(\mathbb{FD}) \to \mathcal{P}_{Pr}(\mathbb{FD})$ by $\mathcal{P}_{Pr}(f)(\mu)(U) = \mu(f^{-1}(U))$. Similar reasoning shows how to extend operators of higher arity. Two facts emerge from this method:

- If we embed $\mathbb{FD}$ into $\mathcal{P}_{Pr}(\mathbb{FD})$ via the mapping $p \mapsto \delta_p$, then the interpretation of each CSP operator on $\mathbb{FD}$ *extends* to a continuous operator on $\mathcal{P}_{Pr}(\mathbb{FD})$: this means that the mapping from $\mathbb{FD}$ into $\mathcal{P}_{Pr}(\mathbb{FD})$ is compositional for all the operators of CSP. This has the consequence that any laws that the interpretation of CSP operators satisfy on $\mathbb{FD}$ still hold *on the image of* $\mathbb{FD}$ *in* $\mathcal{P}_{Pr}(\mathbb{FD})$.
- The way in which the operators of CSP are extended to the model of PCSP forces all the CSP operators to distribute through the probabilistic choice operators. For example, we have

$$a \to (p\,{}_{\lambda}\!+ q) = (a \to p)\,{}_{\lambda}\!+ (a \to q),$$

  for any event a and any processes p and q. This has the result that some of the laws of CSP fail to hold on $\mathcal{P}_{Pr}(\mathbb{FD})$ *as a whole*.

Example 1. Consider the process $(p\,{}_{.5}\!+ q) \sqcap (p\,{}_{.5}\!+ q)$. The internal choice operator $\sqcap$ is supposed to be idempotent, but using the fact that, when lifted to PCSP, the CSP operators distribute through the probabilistic choice operators, we find that $(p\,{}_{.5}\!+ q) \sqcap (p\,{}_{.5}\!+ q) = p\,{}_{.25}\!+ ((p \sqcap q)\,{}_{2/3}\!+ q)$, which means that the probability that the process acts like p is somewhere between .25 and .75, depending on how the choice $p \sqcap q$ is resolved. This unexpected behavior can be traced to the fact that $\sqcap$ distributes through ${}_{.5}+$. One way to explain this is that the resolution of the probabilistic choice in $p\,{}_{.5}\!+ q$ is an internal event, and using the CSP paradigm of *maximal progress* under which internal events happen as soon as possible, the probabilistic choices then are resolved at the same time as the internal nondeterministic one. The processes on either side of $\sqcap$ represent distinct instances of the same process, but because they are distinct, the probabilistic choice is resolved independently in each branch.

Since it is the fact that $\sqcap$ distributes through ${}_{\lambda}+$ that causes $\sqcap$ not to be idempotent, one way to avoid this issue would be to craft a model which forces us to resolve $\sqcap$ first, *before* the probabilistic choices are resolved.

5 Constructing a new model

It follows from the method of construction that the lifting of the operations from $\mathbb{FD}$ to $\mathcal{P}_{Pr}(\mathbb{FD})$ all distribute through the probabilistic choice operators. This is why certain laws from CSP fail in the extension, such as the failure of the extension $\sqcap$ to PCSP to be idempotent. We view the fact that nondeterministic choice is not idempotent on $\mathcal{P}_{Pr}(\mathbb{FD})$ to be problematical. In the example of the last section, for instance, this leads to the unexpected result that there is no precise probability that the process $(p\ {}_{.5}{+}\ q) \sqcap (p\ {}_{.5}{+}\ q)$ acts like p. We now show how to construct a domain Q which supports nondeterministic choice and probabilistic choice, so that the choice operator is idempotent. Moreover, if P is bounded complete, we can construct Q then there is an e-p pair from P into Q.

One approach to defining an idemptotent nondeterministic choice operator might be to search for an alternative method for extending $\sqcap$ to $\mathcal{P}_{Pr}(\mathbb{FD})$. The search is in vain if we also require that the extension be *affine*, since the categorical extension already satisfies this property, and there cannot be two such extensions (because the image of $\mathbb{FD}$ generates $\mathcal{P}_{Pr}(\mathbb{FD})$). So, we seek to extend the construction so as to accommodate another internal choice operator.

Another approach would be to apply an appropriate power domain operator to $\mathcal{P}_{Pr}(\mathbb{FD})$: Indeed, since $\mathcal{P}_{Pr}(\mathbb{FD})$ is coherent, then the classical lower and upper power domains satisfy $\mathcal{P}_L(\mathcal{P}_{Pr}(\mathbb{FD})), \mathcal{P}_U(\mathcal{P}_{Pr}(\mathbb{FD})) \in \mathsf{BCD}$. However, this is not exactly what we want, because, if we use the standard approach to extending the operations from P to $\mathcal{P}_L(P)$ or $\mathcal{P}_U(P)$ in the case P is a probabilistic algebra, we find that the laws we want no longer are valid. For example, for $X, Y \in \mathcal{P}_U(P)$, then $X\ {}_{\lambda}{+}\ Y = \{x\ {}_{\lambda}{+}\ y \mid x \in X, y \in Y\}$, so $X\ {}_{\lambda}{+}\ X = \{x\ {}_{\lambda}{+}\ y \mid x, y \in X\} \neq X$. In general, $X\ {}_{\lambda}{+}\ X$ will be larger than X. To remedy this, we proceed as follows.

Definition 4. *Let P be a probabilistic algebra, and let $X \subseteq P$. We define*

$$\langle X \rangle = \{x\ {}_{\lambda}{+}\ y \mid x, y \in X \ \wedge\ 0 \leq \lambda \leq 1\}.$$

We say that X is affine closed *if $X = \langle X \rangle$. We let $\mathcal{P}_{UA}(P) = \{X \in \mathcal{P}_U(P) \mid X = \langle X \rangle\}$.*

Theorem 2. *Let P be a probabilistic algebra which is also a coherent domain. Then there is a continuous kernel operator*

$$\kappa \colon (\mathcal{P}_U(P), \supseteq) \to (\mathcal{P}_{UA}(P), \supseteq) \text{ given by } \kappa(X) = \bigcap\{Y \in P_U(P) \mid X \subseteq Y\}.$$

Furthermore, $\mathcal{P}_{UA}(P)$ is a continuous, coherent domain which is also a probabilistic algebra. Finally, $\mathcal{P}_{UA}$ extends to a functor $\mathcal{P}_{UA} \colon \mathsf{Coh} \to \mathsf{BDC}$ which is continuous. □

This leads us to consider the domain $\mathcal{P}_{UA}(\mathcal{P}_{Pr}(\mathbb{FD}))$. We have chosen to focus on the probabilistic power domain analogous to the upper power domain because the upper power domain is the power domain of demonic choice, and this is what underlies the (internal) nondeterministic choice in CSP. We build a model "over" $\mathbb{FD}$ using the following:

Theorem 3. *Let P be a bounded complete, continuous domain. Then:*

1. *There is an e-p pair from P to $\mathcal{P}_{Pr}(P)$.*
2. *There is an e-p pair from $\mathcal{P}_{Pr}(P)$ to $\mathcal{P}_{UA}(\mathcal{P}_{Pr}(P))$.*

Moreover, the embedding $e\colon \mathcal{P}_{Pr}(P) \to \mathcal{P}_{UA}(P)$ is a morphism of probabilistic algebras. □

Using these results, we can begin with $\mathbb{FD}$ and generate a bounded complete, continuous domain $\mathcal{P}_{UA}(\mathcal{P}_{Pr}(\mathbb{FD}))$ that also is a probabilistic algebra. This is the model we have been seeking:

Example 2. We show that the domain $\mathcal{P}_{UA}(\mathcal{P}_{Pr}(\mathbb{FD}))$ is a model for PCSP in which internal choice does not distribute over probabilistic choice. We reason as follows. First, using the standard categorical approach, we can extend the interpretation of each CSP operator on $\mathcal{P}_{Pr}(\mathbb{FD})$ to $\mathcal{P}_{UA}(\mathcal{P}_{Pr}(\mathbb{FD}))$, and these extensions all are continuous. Noting that $\mathcal{P}_{UA}(\mathcal{P}_{Pr}(\mathbb{FD}))$ already has an internally defined inf-operation – $(X, Y) \mapsto \kappa(\langle X \cup Y \rangle)$, we then can conclude $\mathcal{P}_{UA}(\mathcal{P}_{Pr}(\mathbb{FD}))$ is a continuous algebra of the same signature as defines the syntax of CSP. Since we can regard CSP as the initial algebra with this signature, it follows that there is a (unique!) algebra homomorphism $[\![\ ]\!]\colon \mathrm{CSP} \to \mathcal{P}_{UA}(\mathcal{P}_{Pr}(\mathbb{FD}))$, and we take this as our semantic map. Actually, this extends to a semantic mapping from PCSP to $\mathcal{P}_{UA}(\mathcal{P}_{Pr}(\mathbb{FD}))$ since the latter is a probabilistic algebra over $\mathcal{P}_{Pr}(\mathbb{FD})$.

To show that internal choice does not distribute over the probabilistic choices in $\mathcal{P}_{UA}(\mathcal{P}_{Pr}(\mathbb{FD}))$, we simply note that we have chosen the internally defined inf-operation on $\mathcal{P}_{UA}(\mathcal{P}_{Pr}(\mathbb{FD}))$ as our interpretation of $\sqcap$, and since this operator is idempotent on all of $\mathcal{P}_{UA}(\mathcal{P}_{Pr}(\mathbb{FD}))$, we conclude that

$$(p\,{}_{.5}{+}\,q) \sqcap (p\,{}_{.5}{+}\,q) = p\,{}_{.5}{+}\,q$$

for any elements of $\mathcal{P}_{UA}(\mathcal{P}_{Pr}(\mathbb{FD}))$ – in particular, this holds for p and q the denotations of processes from PCSP. But then $\sqcap$ cannot distribute through ${}_{.5}{+}$, because we would then have the equality

$$p\,{}_{.5}{+}\,q = (p\,{}_{.5}{+}\,q) \sqcap (p\,{}_{.5}{+}\,q) = p\,{}_{.25}{+}\,((p \sqcap q)\,{}_{2/3}{+}\,q),$$

which would imply that $p \sqcap q = p\,{}_{.5}{+}\,q$, which certainly does not hold, as easy examples show. □

6 Summary

We have focused on an anomaly with the model for probabilistic CSP devised in [9], namely that the interpretation of internal choice on this model is not idempotent. The anomaly is a result of the way in which the operators of CSP are defined on this model. This motivates the construction of an alternative model for probabilistic CSP. We have given a construction of a model using a new power domain – the power domain of affine, compact upper subsets of a coherent continuous domain, and we showed that this family also is a probabilistic algebra. We then showed that, when applied to the model for PCSP from [9], this model satisfies the property that the interpretation of nondeterministic choice is idempotent, and the model also satisfies all the laws of a probabilistic algebra. This is the only model of this type we know of. In particular, the models defined in several of the papers listed in the references (except, of course, that of [6, 7]) seem not to address this issue.

A question we have left unaddressed is what laws our new model satisfies. This is a very important issue, especially given the tradition of algebraic semantics for CSP and its related languages. An obvious example is the external choice operator $\Box$ of CSP, since this operator becomes internal choice if both branches offer the same event.

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