Simulation of Task Graph Systems in Heterogeneous Computing Environments

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Abstract

This paper describes a simulation tool for the analysis of complex jobs described in the form of task graphs. The simulation procedure relies on the PN-based topological representation of the task graph that takes advantage of directly modeling precedence constraints and other characteristics inherent in Generalized Stochastic Petri Nets (GSPN). The GSPN representation is enhanced with enabling functions that govern the sequence of firings of transitions representing execution of tasks. The regulated flow of activity is carried out observing not only precedence constraints but specific allocation heuristics and communication delays. The tool is useful in evaluating different heuristics described by the corresponding implemented algorithm, or using a deterministic timespan given by a Gantt chart.

1. Introduction

Task graphs represent general computation jobs which have been decomposed into modules called tasks that are executed according to some precedence constraints. Task graphs are a well known tool to study performance issues of complex jobs. A direct solution technique for series-parallel task graphs is reported in [1]; an average completion time of the overall job is derived assuming no restrictions exist on the number and architecture of processing units and with no regard to allocation schemes. Execution times of fork-join parallel programs in multiprocessor environments is discussed in [2]. An approach based on multiplication/convolution is applied to Heterogeneous Computing Systems (HCS) at coarse and fine levels of granularity in [3]. Also, in [4] performance prediction of fork-join task graphs is addressed, where the residence times of each task are estimated in terms of service demands and queuing delays; based on these estimations, the task graph is then systematically reduced.

Markov-based solutions of task graph systems have been reported in [5] and [6]; although limited to relatively small task graphs, a Markov-based solution is used for the analysis of scheduling policies in [6]. Since Stochastic Petri Nets (SPN) provide a natural representation of parallelism and synchronization their use spawns applications from individual parallel and concurrent programs to distributed applications and multi-processor systems [7, 8, 9]. SPN's can be used to directly capture the topological information of a task graph and provide a systematic way for applying factors such as processor heterogeneity, allocation schemes, communication costs, and random execution times. Also, a SPN-based solution can be applied to arbitrary graphs which are acyclic but not necessarily series-parallel [10]. SPN-based tools automatically generate Markov models that represent the execution process of complex task graphs where each state is given by the number of tasks executing in parallel. These models are then solved to compute system performance characteristics such as a distribution of the overall completion time.

When the job represented by a task graph is executed on the processing elements of a HCS, estimating the overall completion time becomes an optimization problem involving the mapping of tasks to processors such that completion time is minimized. Mapping tasks to processing units is a hard problem and several heuristics have been proposed in the literature. However, before choosing the most effective heuristic a method must be available for computing an expected completion time and deriving execution distributions for any given task graph, HCS, and allocation heuristic. The methodology reported in [10] to solve complex task graphs using SPN's is not in itself an optimization technique, but it can be used in conjunction with optimization techniques which attempt to search a space of completion time distributions. However, Markov-based numerical solutions are limited to exponential distributions and often involve a large state space.

Consequently, the solution process may be unstable and subject to stiffness problems rendering inaccurate results. Discrete event simulation can use the framework provided by SPN's [11] and circumvent the limitations encountered in the solution of Markov-based models. The work reported in this paper uses the SPN-based topological representation of task graph systems just as in [10] but applies discrete-event simulation to obtain execution time distributions and estimates of the Mean Time to Completion (MTTC) of the jobs represented. Thus a common model based on SPN's is used to drive a discrete event simulation of the overall job. The method can be used to analyze and compare several assignment heuristics given either the algorithm or a Gantt chart of specific assignment cases. To illustrate the use of the tool several assignment heuristics are evaluated and compared.

The next section of the paper introduces the notation and parameters used. Basic concepts on Petri nets are introduced in section three and their application to describe task graphs is given in section four. Section five deals with the simulation methodology. A brief discussion on allocation heuristics is given in section six. The insertion of communication delays is discussed in section six. Simulation algorithms are presented in section seven. Lastly, applications of the tool are discussed.

2. Parameters and Notation

Throughout the paper the following notation is used to describe the simulation tool and related issues.

• a task graph $G(T, E)$ where the vertex set $T = \{T_1, T_2, \ldots, T_k\}$ consists of k tasks which compose some overall job and the edge set E consists of ordered pairs from T which correspond to data or control dependencies. The topology of T is described in detail by the following:

– an in-degree vector $D = [d_1, d_2, \ldots, d_k]$ where d_i is the number of tasks which must complete before T_i may initiate execution.

– an out-degree vector $H = [h_1, h_2, \ldots, h_k]$ where h_i is the number of tasks which are spawned after the completion of T_i.

– a task graph structure $TG[i][j]$, $1 \le i \le k$, $1 \le j \le h_i$ where $TG[i]$ is an array specifying the h_i tasks which are spawned by the completion of T_i; thus, the ordered pair $(T_i, TG[i, j]) \varepsilon E$.

• a $k \times k$ matrix $pkt[i, j]$, $1 \le i, j \le k$ where $pkt[i, j]$ is the average number of data packets of standard size that is sent from T_i to T_j. Alternatively, these can be specified as edge weights for the elements of E.

• a priority vector $W = [w_1, w_2, \ldots, w_k]$ which induces a sequential ordering of any ready tasks assigned to the same processor; these priorities may be taken from the indices of the tasks, e.g. $w_i = k - i$, or they may be randomly or determined according to the assignment heuristic employed.

• a set $P = \{P_1, P_2, \ldots, P_n\}$ consisting of n processors composing a heterogeneous suite.

• a $k \times n$ execution time matrix $B[i, j]$, $1 \le i \le k$, $1 \le j \le n$ where b_{ij} is the average execution time of T_i on P_j.

• an $n \times n$ communication time matrix $C[r, s]$, $1 \le r, s \le n$ where each entry c_{rs} is the average communication time to transfer a data packet of standard size from P_r to P_s.

• a $k \times n$ static allocation matrix $A[i, j]$, $1 \le i \le k$, $1 \le j \le n$ where entry $a_{ij} = 1$ if T_i has been allocated to P_j, and 0 otherwise.

3. Basic Petri Net Concepts

A Petri net (PN) is a directed, weighted, and bipartite graph [12]. PN's are bipartite in that nodes are of two types, *places* and *transitions*, with arcs occurring either from places to transitions or from transitions to places. When an arc is from a place p to a transition t, then p is an input place of t; a place p is an *output place* of t if an arc proceeds from t to p. Places and transitions are represented pictorially by circles and thin rectangles, respectively. A third component of any PN are tokens which reside in places; pictorially, tokens are represented by dots within the perimeters of places. Tokens are transferred from one place to another by the firing of transitions. When a transition t fires, tokens are removed from all input places of t and placed in the output places of t; thus, enforcing a logical flow of activity throughout the net. A transition can fire if it is enabled, i.e., if all of its input places possess at least one token. An arc may be weighted where the weight specifies the number of tokens which must reside in an input place in order for a transition to be enabled, or the number of tokens placed in an output place by the firing of an enabled transition; if the weight is unspecified then it is assumed to be one. PN's and their dynamic behavior can be captured in mathematical notation via state vectors. Given a PN with k places, a marking q of the PN is denoted by M_q; a marking is described by a k – vector whose ith component denotes the number of tokens in place p_i; an initial marking of the PN is denoted by M_0. A particular PN with an underlying graph N is denoted (N, M_0). The reachability graph of a PN is a graph $G_R(M, \Delta)$ where the vertex set M is the set of all possible markings for the PN and the edge set Δ consists of all possible transition firings transforming one marking into another.

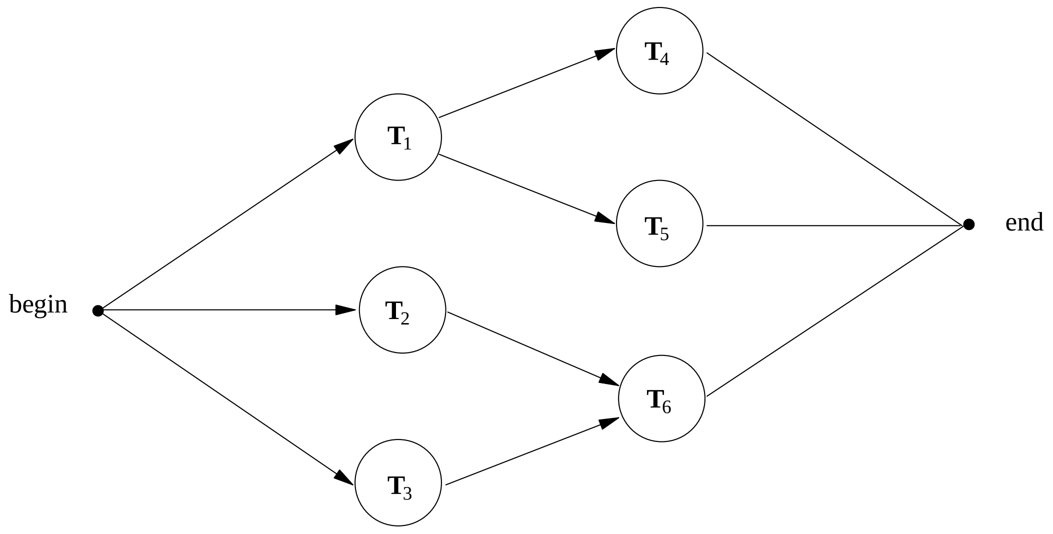


Figure 1. A simple task graph

Stochastic Petri nets are PN's in which there is an exponentially distributed delay time between the enabling and firing of transitions. The reachability graph of a bounded SPN is isomorphic to a finite Markov chain (MC) [13]; in particular, the markings of the reachability graph comprise the state space of a MC and the transition rate between any two states X_i and X_j is the sum of all firing delays for transitions transforming M_i into M_j. *Generalized stochastic Petri nets* (GSPN) have been proposed [14] in which transitions are of two types: *timed* transitions which have the exponentially determined firing rates and *immediate* transitions which have no firing delay and have priority over any timed transition. *Enabling functions* are marking-dependent functions which can be defined on each transition as a switching mechanism. Transition priorities (timed vs. immediate) and enabling functions are logically equivalent extensions of SPN which endow them with the full computational power of Turing machines [15]. In this paper the notion of GSPN is used.

4. GSPN Models of Task Graphs

Task graphs are assumed to be series-parallel for several approaches to performance evaluation [16] and optimization [17]; however, this limitation is avoided in the PN-based methodology of this work. Fig. 1 shows a simple task graph which will be used to illustrate the translation of task graphs into GSPNs. The translation of a task graph into a GSPN begins with the association of each task T_i with a place/timed transition pair, p_i and t_i. Fig. 2 shows the GSPN corresponding to the task graph in Fig. 1. Auxiliary places xp_0 and xp_1 and immediate transitions it_0 and it_1 are used to enforce initiation and completion conditions, respectively, for the overall job. The presence of at least one token in a place may represent the fulfillment of all preconditions for the initiation of the task. The firing of a timed transition represents the completion of execution of the corresponding task. The delay time of each transition corresponds to the exponentially distributed execution time of the task. A place p_i can be associated with the in-degree d_i to enforce precedence constraints. Initially, the presence of a token in xp_0 enables it_0; the firing of it_0 represents the initiation of an execution cycle. The presence of three tokens in xp_1 and the firing of it_1 indicates that an execution cycle has been completed. Timed transitions in the GSPN model in Fig. 2 will fire once enabled. Beginning with an initial marking M_0 a sequence of markings can be generated to form a reachability graph. The set of markings generated correspond to the possible execution states of the system, where a system state is defined by the tasks which are executing concurrently. If firing times are exponentially distributed the set of markings generated corresponds to a Markov chain that can be solved using well known tools such as SPNP [18] or SHARPE [1].

Consider some marking M_i in which task T_6 should be ready to run. To make this possible, both T_2 and T_3 must have finished execution; this will be indicated by the presence of two tokens in p_6, i.e. $x_i(p_6) = 2$. To capture this precedence constraint it suffices to associate each input arc into a timed transition with a weight corresponding to the in-degree of each node in the task graph. Alternatively, the in-degree vector is associated with marking-dependent enabling functions.

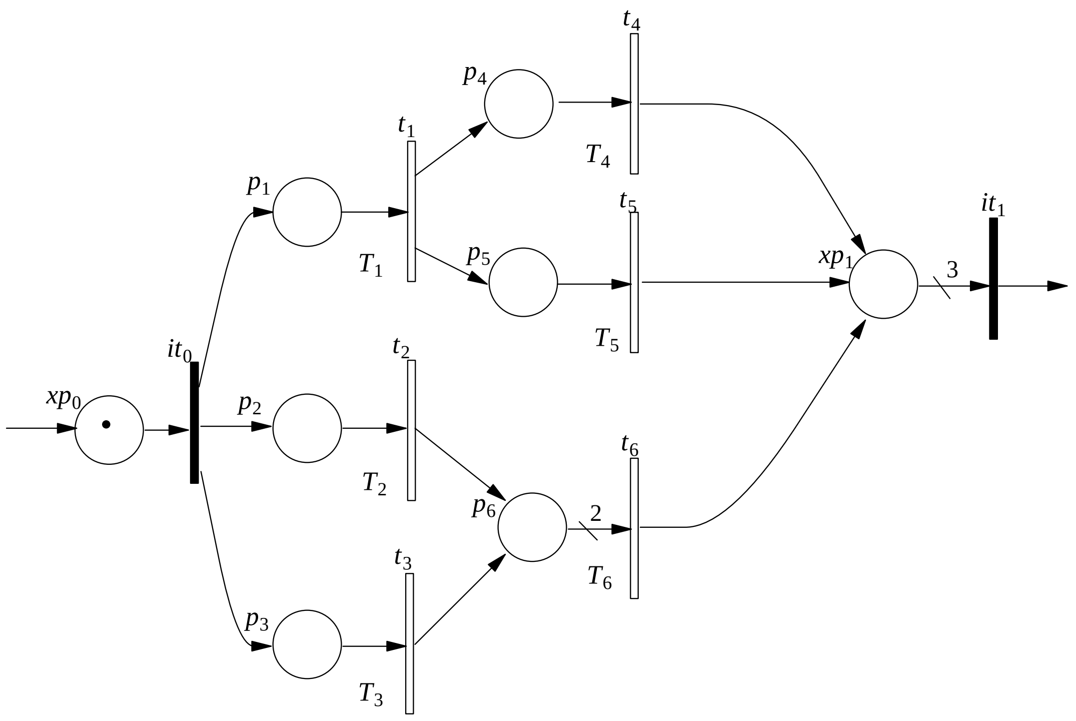


Figure 2. GSPN model of the task graph from Fig. 1

5. Simulation Methodology

In a SPN model, the firing of transitions represents the occurrence of events, in this case, execution of tasks. To simulate execution of tasks [11], a clock is set for each newly enabled transition to keep track of the execution time until the transition fires. The simulation procedure must also check for precedence constraints, availability of processors, and priority of tasks. When a transition is enabled its firing time is generated as a random variate from a selected distribution. Firing times are recorded by associating clocks to transitions. The PN-based simulation procedure takes place observing the following major steps:

1) Check for newly enabled transitions,

2) Generate firing firing times, and

3) Update clocks.

5.1. Enabling Functions

A transition t_i is enabled when conditions Q_i, V_i, and Z_i are satisfied. An entry of the enabling vector $F = [\, f_i], 0 \le i \le k$ is evaluated such that if:

$$f_i = Q_i V_i Z_i$$

evaluates to one and t_i can fire.

Condition Q_i checks for precedence constraints, that is, when the number of tokens m_i in place p_i is equal to the in-degree d_i of the vertex representing task T_i, its precedence constraints are met, i.e.,

$$Q_i = \begin{cases} 1 & \text{if } \; m_i = d_i \\ 0 & \textit{otherwise} \end{cases}$$

Condition V_i checks for allocation and availability of processors. To check for allocation suffices to examine the *ith* row of matrix A for $a_{ij} = 1$ and then verify if processor j is free. Let a binary vector $FREE = [\, free_j], 0 \le j \le n-1$ keep track of which processors are currently free, then

$$V_i = a_{ij}\, free_j$$

If more than one transition satisfies condition Q and V and their corresponding tasks are allocated to the same processor, only one transition should be enabled (only one task should execute) even though these tasks could execute in parallel. The task with the highest priority is chosen using the priority vector W. Let Rdy be the set of transitions representing parallel tasks allocated to the same processor. That is, the set of transitions that could be enabled from a current marking M, then

$$Z_i = \begin{cases} 1 & \text{if } w_i = \max\limits_{j\varepsilon Rdy}\{w_j\} \\ 0 & otherwise \end{cases}$$

Note that these functions could be easily implemented by incorporating additional places and transitions to the model in Fig. 2. For example the presence of a token in a dedicated place can be used to model the availability of a processor and to derive statistical measurements on the usage of that processor [19]. Also additional immediate transitions can be used to model task priorities. It can be argued that additional modeling elements may obscure the representation of a task graph and although they are useful, they become transparent to the user when dealing with large complex models. We find the addition of places to model processing elements and their interconnections useful for the case of analyzing the behavior of systems running several jobs modeled by different task graphs or several instances of the same job in an effort to capture the load of the system, resource contention and usage. In our case the effect of external load is reflected in the execution time of each subtask. The use of enabling functions keeps the model simple and the simulation code relatively simple as well.

5.2. Firing times

If a timed transition is enabled, a firing time is generated using a firing transition rate given in terms of the average execution times of each task obtained from matrix B. Random variates are generated from three possible distributions: exponential, normal, and uniform. The values given by matrix B are used according to the distribution function selected. Uniform and normal functions require a second value that must be provided by the user. If $B1$ denotes the first matrix given as the execution matrix B then $B2$ denotes a second matrix provided by the user for the case of normal and uniform distributions. For exponential and normal distributions $b1_{ij}$ provides the average execution time. For normal distributions the matrix $B2$ provides the standard deviation ρ_{ij}. In the case of a uniform distribution, matrix $B1$ provides the starting point $b1_{ij}$ and matrix $B2$ provides the ending point $b2_{ij}$. These values are used to calculate the mean as $(b1_{ij} + b2_{ij})/2$. A pseudo-random number u is generated from $U(0, 1)$. A firing time x_{ij} associated to transition t_i is generated for each distribution as follows:

i). Exponential distribution, $\exp(b1_{ij})$:

$$x_{ij} = -\, b1_j \times \ln(u)$$

ii). Normal distribution, $N(b1_{ij}, b2^2_{ij})$:

$$x_{ij} = (\sum_{a=1}^{a=12} u_a - 6) \times b2_{ij} + b1_{ij}$$

iii). Uniform distribution, $U(b1_{ij}, b2_{ij})$:

$$x_{ij} = u \times (b2_{ij} - b1_{ij}) + b1_{ij}$$

5.3. Clock Update

A local clock that keeps track of firing times and a global clock is used to record the overall completion time. When a timed transition is enabled a local clock is set to the generated firing time to indicate the remaining time until the transition fires. A global clock is denoted as C and local clocks are represented by a vector $LC = [lc_i]$, $0 \le i \le k-1$ where lc_i is the local clock associated to transition t_i.

Since local clocks indicate remaining times, they are discarded when they reach 0 time units and the corresponding transitions fire. At the moment a transition fires, the global clock and local clocks are updated. The global clock update is performed by adding the minimum local clock time min_t to the global clock C; min_t is taken from the set of enabled transitions that have not yet fired. The following expressions are used to update all clocks.

$$C = C + min\_t$$

$$lc_i = lc_i - min\_t$$

where $min\_t = \min\limits_i \{lc_i\}$, $0 \le i \le k-1$. Once the last transition fires, the global clock C indicates the overall completion time.

6. Heuristics

Different allocation heuristics can be evaluated by mapping them into the allocation matrix A. To illustrate the use of the simulation methodology discussed in this paper four static allocation heuristics are evaluated and compared.

1. *Shortest Estimated Execution Time First (SEETF).* In this scheme [20, 21, 22] task T_i is selected at random from the task set and assigned to the processor that executes T_i faster. The elements of the task allocation matrix from the SEETF algorithm are determined as follows:

$$a_{ij} = \begin{cases} 1 & \text{if } b_{ij} = \min\limits_j \{b_{ij}\} \\ 0 & otherwise \end{cases}$$

2. *Minimum Finish Time (MFT).* In this allocation scheme [22], task T_i is also selected randomly from a topologically sorted task set, i.e. taking into account the precedence constraints between tasks. The selected processor is the one that minimizes the finish time of a task in a deterministic simulated execution, where the finish time of a selected task T_i is given by the

minimum sum of its execution time b_{ij} and the next time instance in which processor P_j becomes a free processor.

$$a_{ij} = \begin{cases} 1 & \text{if } \min_j \{b_{ij} + \text{ time until } P_j \text{ is free}\} \\ 0 & \text{otherwise} \end{cases}$$

Note that all tasks are selected randomly but restricted to those tasks whose predecessors have already been allocated.

3. *Largest Task First (LTF)* [22]. The selection of tasks is based on service demands. The task with the largest service demand is selected first, or alternatively the task with the largest execution time is selected first. Thus:

$$a_{ij} = \begin{cases} 1 & \text{if } b_{ij} = \max_i \{b_{ij}\} \\ 0 & \text{otherwise} \end{cases}$$

A processor P_j is selected randomly.

4. *Most Data Task First (MDTF)* This scheme selects the task that generates most data. The data generated by a task T_i is determined in terms of the number of data packets going out, that is:

$$pkt_i = \sum_{j=1}^{k} pkt_{ij}$$

Thus, the construction of the allocation matrix proceeds as follows:

$$a_{ij} = \begin{cases} 1 & \text{if } pkt_{ij} = \max_i \{pkt_i\} \\ 0 & \text{otherwise} \end{cases}$$

and in this case also the processor P_j is selected randomly.

7. Communication Delays

As in [10], two approaches are presented based on two types of interconnection networks: (a) a high-performance network characterized by high-connectivity and parallel communications and (b) a bus-oriented network with low-connectivity. In both cases, output data is assumed to be accumulated in a buffer during task execution and transmitted after task completion.

7.1. Modeling High-performance Communication Networks

High-performance communication networks can be characterized as expensive systems in which inter-node communication takes place on dedicated, point-to-point links. Data intended for each successor is written to a separate buffer. Furthermore, each processor may be coupled with a front-end communication processor which enables parallel communication. In terms of a task graph, once a given task completes, successor tasks experience an initiation delay equal to the data transfer time for all intended packets; ideally, any successor task allocated to the same processor as the parent task should be able to begin execution immediately after the completion of the parent task.

The properties of such a high-performance network can be modeled in a GSPN by inserting additional place/timed-transitions to represent each individual communication; augmentation of the task graph with communication nodes has been proposed for CTMC-based analysis [23] and at the SPN level [24]. Each timed-transition inserted is associated with an exponentially distributed delay whose parameter is the average communication time between the host processors. Thus, given a completed task T_i allocated to processor P_r and a successor task T_j allocated to P_s, the average communication rate assigned to the transition modeling the transfer of data is given by:

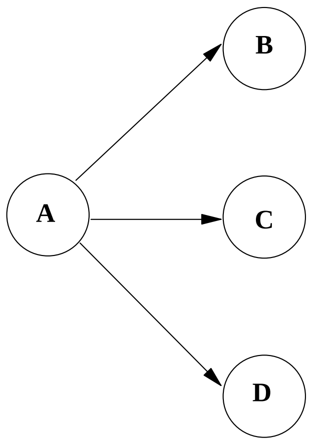
$$\delta_{ij} = \frac{1}{c_{rs}\, pkt_{ij}}$$

Fig. 5a illustrates a segment of some task graph in which Task A spawns tasks B, C, and D. Suppose the four tasks are allocated to three processors such that A and C are allocated to one processor, and B and D are allocated to the other two processors, then the resulting GSPN for Case 1 would be as shown in Fig. 5b. Note the insertion of place/transition pairs between A and B and A and D to represent the individual

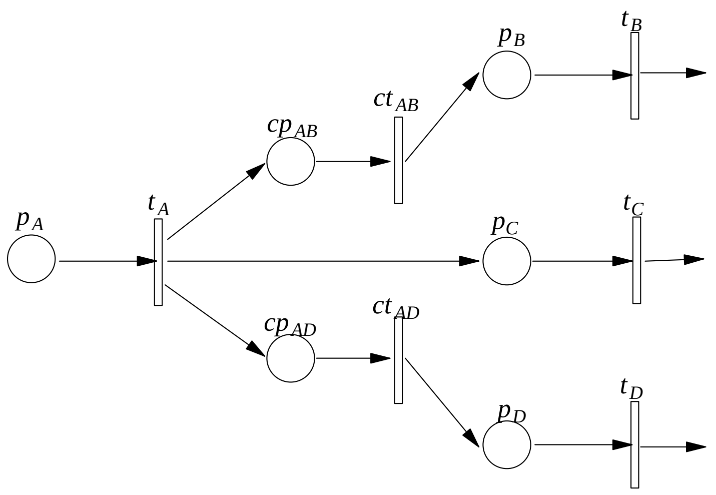
In terms of simulation, communication delays are determined from a distribution function using the average delay δ_{ij} and associating a local time to communication tasks.

7.2. Modeling Bus-Oriented Networks

In interconnection networks characterized by low-connectivity, groups of processors may have to share common communication links, as is the case with a bus-oriented architecture. Also, in lower cost systems processors may be forced to expend computation cycles on communication processing. If, additionally, output data packets for successor tasks are queued up in a single buffer in some random ordering and transmitted on a FIFO basis, then it is highly unlikely that a successor task will receive all of its packets before any other successor task. In terms of the example in Fig. 5a, if the processor to which task A is allocated must broadcast packets in random order to the processors associated with tasks B, C, and D , then it is reasonable to assume that on average B, C, and D will experience uniform initiation delay.



a) Segment of a task graph



b) SPN with communication nodes

Figure 5. GSPN model assuming a high-performance network

Such behavior can be reflected in the GSPN by simply modifying the rate function governing the firing of the transitions associated with each task. In this case, no extra nodes are inserted in the PN model. Rather, the firing delay of each transition is increased by the sum of communication costs associated with each successor task. Let T_i be allocated to P_j where completion of T_i spawns $m = h_i$ tasks $T_{q_1}, T_{q_2}, \ldots, T_{q_m}$ which are allocated to processors $P_{y_1}, P_{y_2}, \ldots, P_{y_m}$. Then a modified firing rate for transition t_i is given by:

$$\tilde{\lambda}_i = \frac{1}{\mu_{ij} + \sum_{k=1}^{m} c_{jy_k} pkt_{iq_k}}$$

This new value is then used to determine execution times from the distribution function of choice with the value of μ_{ij} determined accordingly. In reality a given network may be heterogeneous with respect to interconnection capabilities. In this case the GSPN model can be systematically constructed to appropriately model each segment of the network, reflecting the different sets of assumptions mentioned above. The net result is that the simulation process uses a GSPN representation with dynamically determined transition rates and enabling functions capturing the full interplay of task precedence relationships, allocations specifications, availability of idle processors, diverse execution rates across a heterogeneous suite, and communication delays.

8. Simulation Algorithms

A simulation algorithm based on the PN-based topological description of task graphs is now described. The algorithm generates the MTTC and a tabulation to plot the cumulative probability distribution of the execution time. The following steps summarize the simulation process for the case in which no communication delay is taken into account:

1) Initialize the global clock C and the initial marking M_0.

2) Check for newly enabled transitions. In the absence of newly enabled transitions go to step 5).

3) For each enabled timed transition t_i generate firing time x_{ij}.

4) For each enabled timed transitions, set the local clock lc_i to $lc_i = x_{ij}$.

5) Find the minimum local clock *min_t*

6) Fire the transition with the minimum clock *min_t*. Once a timed transition fires, the corresponding task completes execution and the host processor is released.

7) Update global clock C and local clocks lc_i. Notice that by firing transitions with the minimum remaining time equal to *min_t*, its $lc_i = 0$ and removed from the set of lc_i's. The firing of the last timed transition ends the current cycle. A new cycle begins at step 1) by resetting the initial marking M_0 and the global clock C.

8) Update the marking record and repeat from step 2).

The above procedure is also used for the case of low-performance networks where the firing rates are modified accordingly. To take into account transfer delays in a high-performance network some modifications are needed. Let cc_{ih} denote the communication clock between task T_i and task T_h. Note that transition t_i is a transition that has already fired, that is, the corresponding task T_i is in the process of transferring data. After transfer is complete, a token travels to output place p_h. The set of communication clocks cc_{gh} is also compared with local clocks lc_i to determine the minimum time min_t. Note that the set of lc_i's corresponds to transitions t_i that have been enabled but are not yet transferring data. If the min_t selected corresponds to a local clock lc_i, then transition t_i fires, else, the min_t corresponds to a communication clock and a token is now transferred to a destination place p_h. Steps 1) to 4) are the same and the rest of the algorithm is modified as follows:

5) Find the minimum local clock: $min\_t = \min_{ih} \{lc_i, cc_{ih}\}$.

6) Update global clock C, local clocks lc_i, and communication clocks cc_{ih}:

$$C = C + min\_t$$

$$lc_i = lc_i - min\_t$$

$$cc_{ih} = cc_{ih} - min\_t$$

7) If min_t corresponds to a local clock lc_i, then:

7.1) Transition t_i fires. Tokens are removed from the input places and the corresponding processor is released.

7.2) If t_i is the last transition, then stop the cycle.

7.3) Generate communication delays and set communication clocks to $cc_{ih} = \frac{1}{\delta_{ih}}$.

8) If min_t corresponds to a communication clock then transfer a token to output place p_h.

9) Update the marking record and go back to step 2).

9. Applications

A hypothetical 13-node task graph is shown in Fig. 6. This graph was used in [10] to illustrate a PN-based numerical approach to the solution of complex task graphs. The simulation procedure is applied to the task graph and compared with the results rendered by the SPNP tool [18]. The static allocation scheme used maps tasks to processors such that a task T_i is assigned to processor P_j where $j = i \, mod n$. The edge weight shown in Fig. 6 correspond to the number of standard sized packets generated and sent to successor tasks.

The following matrix B specifies the spectrum of execution times for each task across six processing units in the system in standard time units per execution:

$$B^T = \begin{bmatrix} .9 & 2 & .3 & 1 & .3 & 4 & 2 & 1 & 3 & 2 & .3 & .2 & .1 \\ .3 & 4 & .3 & 1 & .3 & 5 & 2 & 1 & 3 & 4 & .5 & .5 & .1 \\ .5 & 1 & .5 & 1 & .3 & 5 & 2 & 1 & 4 & 2 & 5 & .2 & .3 \\ .5 & 2 & .2 & 2 & .3 & 5 & 2 & 2 & 2 & 2 & .5 & .2 & .1 \\ .5 & 2 & .3 & 1 & .6 & 5 & 3 & 1 & 3 & 2 & .5 & .2 & .1 \\ .5 & 2 & .3 & 1 & .3 & 7 & 1 & 1 & 3 & 2 & .3 & .2 & .1 \end{bmatrix}$$

The communication delays per data packet in the interconnection network between the six processors are characterized by the matrix C in terms of standard time units per packet:

$$C = \begin{bmatrix} 0 & .1 & .1 & .2 & .2 & .1 \\ .1 & 0 & .4 & .3 & .2 & .1 \\ .1 & .4 & 0 & .2 & .3 & .3 \\ .2 & .3 & .2 & 0 & .3 & .2 \\ .2 & .2 & .3 & .3 & 0 & .1 \\ .1 & .1 & .3 & .2 & .1 & 0 \end{bmatrix}$$

Relative priorities among the 13 tasks are specified thus:

$$W = [13\ 12\ 11\ 8\ 9\ 10\ 7\ 6\ 5\ 4\ 3\ 1\ 2]$$

It should be noted that this priority scheme is entirely arbitrary as is the allocation scheme. The numerical and simulation results shown in Fig. 7 correspond to the probability of completion at time t, $P(X \le t)$ of the overall job based on three communication scenarios: a) there are no communication costs, b) communication occurs over a high- performance network, and c) communication takes place over a low-performance network. The $MTTC$ results along with confidence intervals are given in Table 1. Up to 1000 task graphs were simulated and the time to render averaged results took about 1.69 secs. compared with 125.13 secs. needed by the numerical tool (SPNP) in a Sparc classic workstation. This difference is in part due to the large number of states generated. For the case of the low performance network, SPNP took 2.40 secs. while the simulation process took 0.63 secs [25].

A second application consists in evaluating the task graph shown in Fig. 8. This 20-node task graph describes the LU decomposition algorithm common in the solution of linear systems encountered in many scientific applications. Several schedules for different heuristics were derived in [26]. Two heuristics the Heavy Node First (HFN) and Weighted Length (WL) were examined to determine the corresponding assignment matrices A_{HNF} and A_{WL}, respectively:

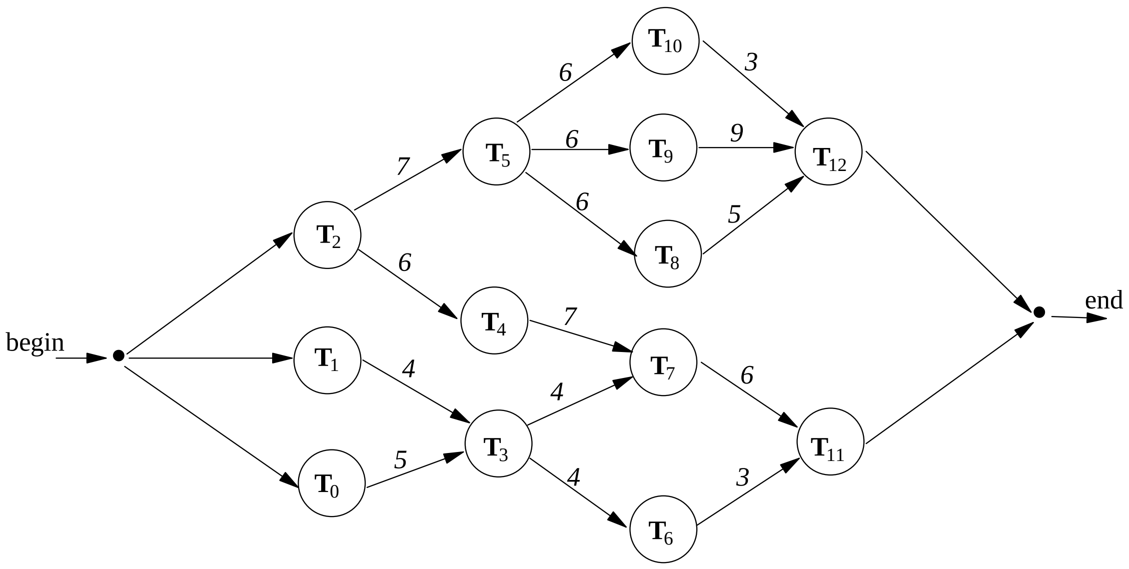


Figure 6. A 13-node complex task graph

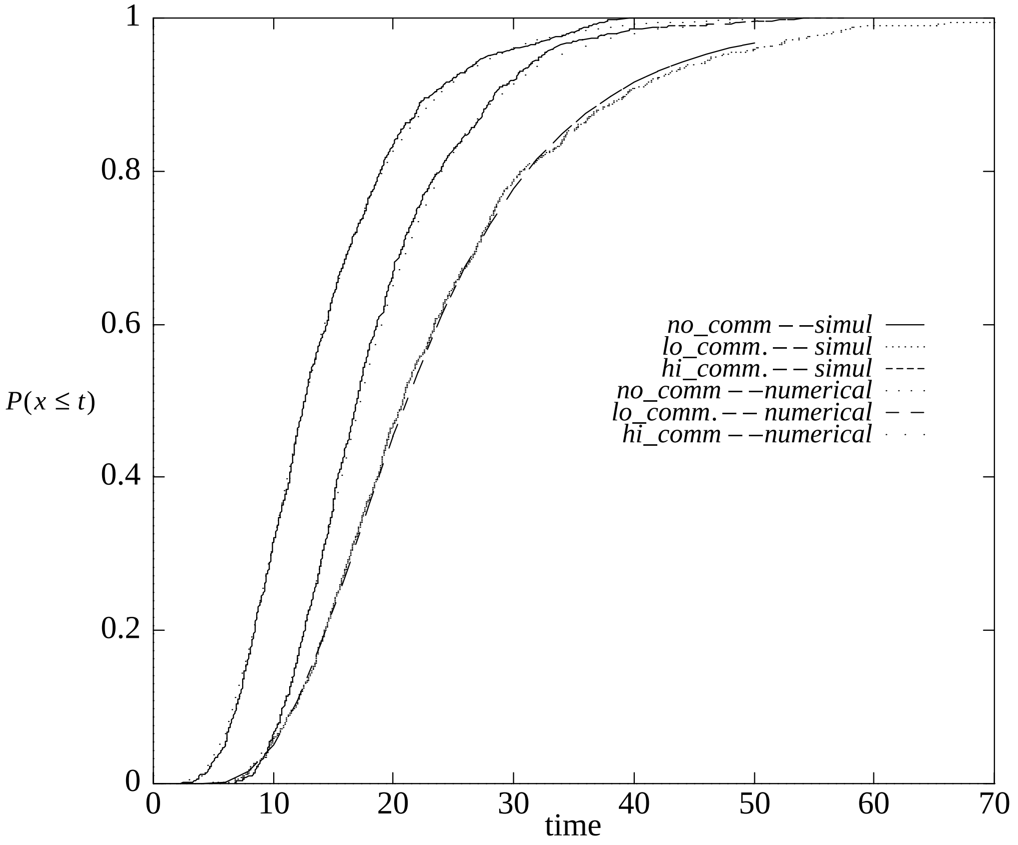


Figure 7. CDF of completion time given static allocation and network type

Table 1. Comparison of MTTC results

Case	Numerical MTTC	Simulation	
		MTTC	99% confidence intervals
High-Performance Network	18.8269	18.5959	17.9854 – 19.2063
Low-Performance Network	23.5204	23.5772	22.6119 – 24.5426
No-communication Costs	14.1999	14.1491	13.5817 – 14.7165

$$A_{HNF}^{T} = \begin{bmatrix} 1\;0\;0\;0\;1\;0\;1\;0\;0\;0\;0\;0\;0\;1\;1\;0\;0\;1\;0\;0 \\ 0\;1\;1\;0\;0\;1\;0\;0\;0\;0\;1\;0\;0\;0\;0\;1\;0\;0\;1\;0 \\ 0\;0\;0\;1\;0\;0\;0\;1\;1\;1\;0\;1\;1\;0\;0\;0\;1\;0\;0\;1 \end{bmatrix}$$

$$A_{WL}^{T} = \begin{bmatrix} 1\;0\;1\;0\;0\;0\;0\;1\;1\;0\;0\;1\;1\;0\;0\;0\;1\;0\;0\;1 \\ 0\;1\;0\;1\;0\;1\;0\;0\;0\;0\;1\;0\;0\;0\;1\;1\;0\;0\;1\;0 \\ 0\;0\;0\;0\;1\;0\;1\;0\;0\;1\;0\;0\;0\;1\;0\;0\;0\;1\;0\;0 \end{bmatrix}$$

Both heuristics are based on the execution times (weights) of each task. The HNF heuristic examines the task graph level by level assigning the heaviest nodes first. The WL heuristic assigns control nodes first by associating a rank determined in terms of the length of an exit path, branching factor, number of depending tasks in the path and their weights. For further details see [26]. The schedules reported in the form of Gantt charts were derived assuming the following:

1) The processing units are identical,

2) A communication over processing time ratio very high. Consequently, communication delays are assumed negligible, and

3) Execution times as shown in Fig. 7.

The simulation of these two heuristics under a uniform distribution with zero variance rendered the same total execution time of 96 units. Again examining the schedules the following priority vectors were obtained:

$$W_{HNF} = [20\ 19\ 16\ 15\ 14\ 13\ 8\ 9\ 12\ 18\ 7\ 3\ 6\ 11\ 17\ 4\ 2\ 5\ 10\ 1]$$

$$W_{WL} = [20\ 19\ 17\ 16\ 15\ 11\ 8\ 10\ 14\ 18\ 7\ 3\ 6\ 12\ 13\ 4\ 2\ 5\ 9\ 1]$$

Thus, any instance of heuristics given in the form of Gantt charts such as those derived to compare declustering techniques in [27] can be similarly characterized for the simulation procedure. Fig. 9 shows the plots obtained in the evaluation of SEETF, MFT, LTF, HNF, and WL heuristics under the assumption of exponentially distributed execution times. The priority vectors for the first three schemes were derived as the assignments were made. The results show that HNF and WL perform better as expected. Again 1000 copies of the task graph were used in the simulation.

10. Conclusions

The numerical solution of task graphs based on a GSPN model is limited to execution times that are exponentially distributed. A reliable evaluation of large complex task graphs is not guaranteed as it involves the solution of an underlying very large state space. One way to circumvent this problem is using simulation. The simulation technique discussed relies on a the PN-based topology of a given task graph. Besides naturally capturing the dynamics of a job execution, another advantage in relying on a PN-based topology is that a common model is used for both a numerical and a simulation-based analysis. This is useful in the development of a user interface currently under construction that incorporates both methods of solution.

The simulation tool presented facilitates the analysis and comparison of allocation heuristics. This is illustrated by the evaluation of four heuristics for a particular application. Results are reported to compare the behavior of two types of networks and a comparison is made between simulation results and those obtained using a numerical evaluation. The results of these comparisons validate the simulation tool implemented. It turns our that the simulation algorithm implemented is faster than the numerical solution of the cases reported because of the largeness problem. However, an interface currently under development is required to handle large applications involving thousands of tasks. Also, the tool can be used to explore and determine optimal size of networks in terms of the number of processors to achieve the best performance of a particular application. Since simulation avoids the problem of state explosion present in Markov-based models, a useful extension to this tool must include the analysis of multiple task graphs for which the use of color Petri nets would be more suitable.

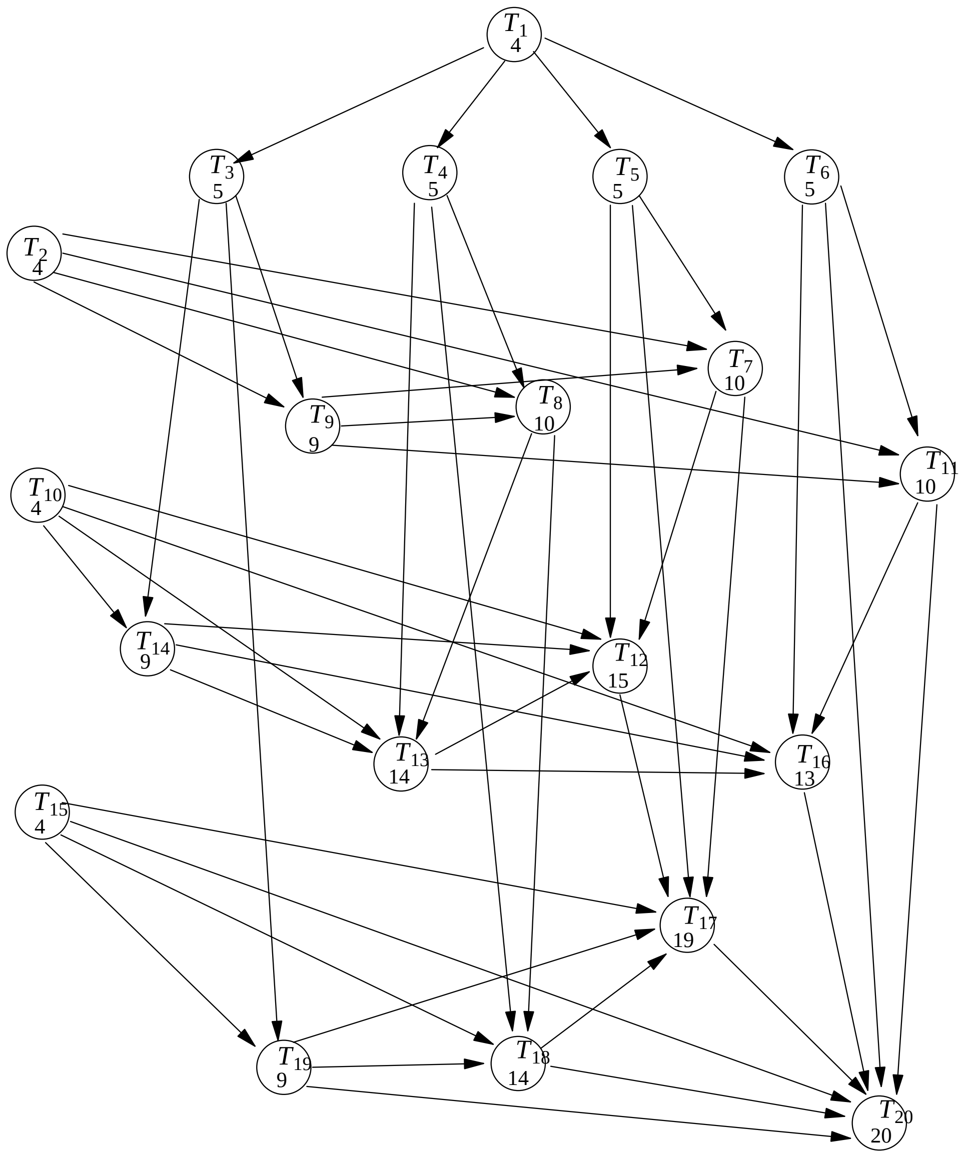


Figure 8. Task Graph for the LU decomposition algorithm

11. Acknowledgements

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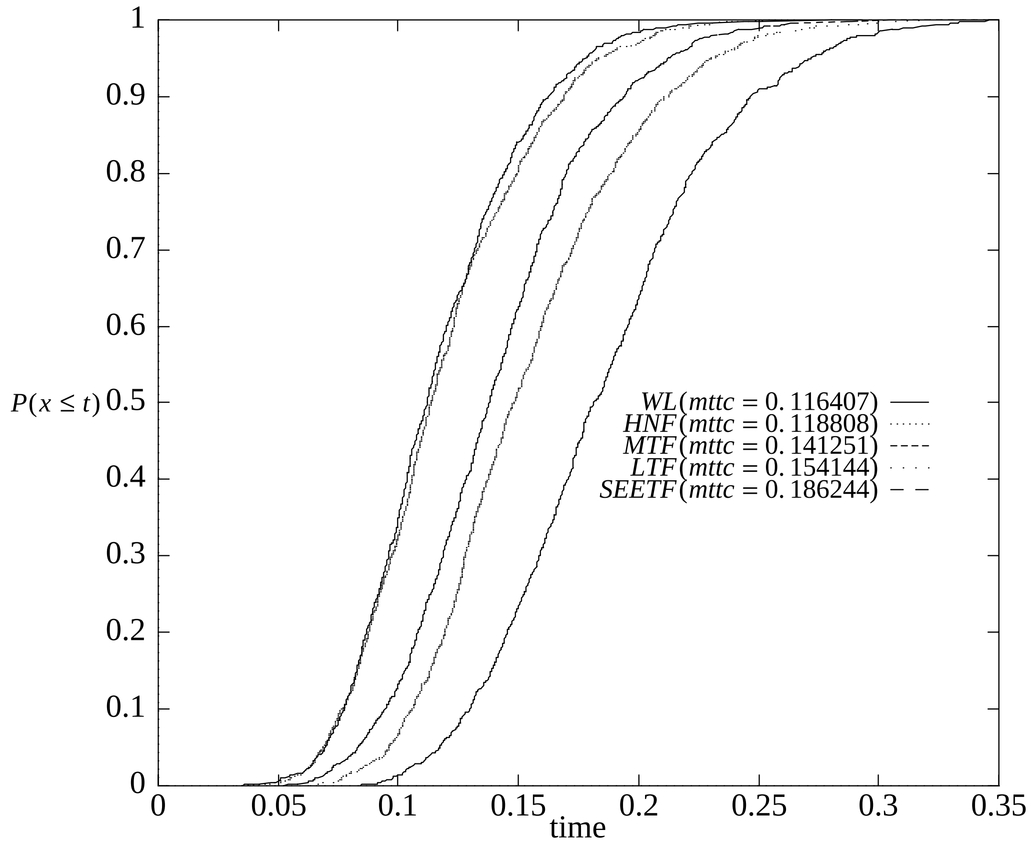

Figure 9. CDF of completion time for the LU decomposition algorithm

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