Multicasting and Broadcasting in Large WDM Networks

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Abstract

We address the issue of multicasting and broadcasting in wide area WDM networks in which a source broadcasts a message to all members in S ($\subset V$). We formalize it as the optimal multicast tree problem which is defined as follows. Given a directed network $G = (V, E)$ with a given source s and a set S of nodes, $|V| = n$ and $|E| = m$. Associated with every link $e \in E$, there is a set $\Lambda(e)$ of available wavelengths on it. Assume that every node in S is reachable from s, the problem is to find a multicast tree rooted at s including all nodes in S such that the cost of the tree is the minimum in terms of the cost of wavelength conversion at nodes and the cost of using wavelengths on links. That is, not only do we need to find such a tree, but also do we need to assign a specific wavelength $\lambda \in \Lambda(e)$ to each directed tree edge e and to set the switches at every node in the tree. We show the problem is NP-complete, and hence it is unlikely that there is a polynomial algorithm for it. We further prove that there is no polynomial approximation algorithm which delivers a solution better than $(1-\epsilon')\ln n$ times the optimum unless there is an $n^{O(\log\log n)}$ time algorithm for NP-complete problems, for any fixed ϵ' with $0 < \epsilon' < 1$. We finally reduce the problem to a directed Steiner tree problem on an auxiliary directed graph. As results, any approximation solution for the directed Steiner tree problem gives an approximation solution for our problem with the same degree accuracy, i.e, there is an approximation algorithm for the problem which runs in time $O(k^2 n + km + kn\log(kn) + (kn)^{1/\epsilon}|S|)$, and delivers a solution within $O(|S|^\epsilon)$ the optimum for any fixed ϵ with $0 < \epsilon \leq 1$. Moreover, we also present a distributed algorithm for the problem. The communication and time complexities of the distributed algorithm are $O(km)$ and $O(kn)$ respectively, and the solution delivered is $|S|$ times the optimum, where k is the number of wavelengths in the network.

1 Introduction

Multicast communication involves the transport of information between a single sender (source) and multiple receivers (destinations). A special case of multicast is *broadcast* where the set of receivers consists of all other nodes in the network except the source. Multicast applications includes video conferencing, entertainment distribution, tele-classrooms, distributed data processing, etc. With growing demand for these services and the availability of high bandwidths, future networks such as WDM networks must be equipped to handle multicast communication in an efficient manner. The most popular solutions to multicast routing involve tree construction. Algorithms for constructing multicast trees have been developed in traditional *undirected* networks [11, 12, 1, 10, 16, 7]. Some of these algorithms have multiple optimization objectives for the tree such as average delay [11], maximum delay [12], delay variation-bounded [16], etc. Some even considers the update version of the problem by taking into account adding or deleting members from the tree [1]. It must be mentioned that there is a big difference between the traditional networks and the WDM networks regarding the multicasting issue. This difference reflects in their optimization objectives. For example, in traditional networks we usually focus on minimizing the total of bandwidth spent and the maximum transfer delay, etc. However, in WDM networks the available wavelengths are crucial resources, and the cost for wavelength conversion at some nodes (switching from one wavelength to another wavelength) is an important factor.

In this paper we first formalize the optimal multicast tree problem in WDM networks as follows. Assume that there are a given source s and a set S of destinations in the network such that every node in S is reachable from the source. The objective is to find a multicast tree rooted at the source including all nodes in S such that the cost sum of links and nodes in the tree is minimum in terms of the following cost measurement: 1) the cost for traversing a link on some wavelength; and 2) the cost for wavelength conversion when the path has to switch to a different wavelength at some intermediate nodes. Note that this cost measurement was defined by Chlamtac et al [3] for finding an optimal lightpath/semilightpath between a pair of nodes in the network. Clearly the problem is more complicated than its counterpart in traditional networks. We then show the problem is NP-complete. So, it is unlikely that there is a polynomial algorithm for finding an exact solution for the problem unless P=NP. Instead, we shall look for an approximation solution for it. However, we further show that there is no polynomial approximation algorithm which delivers a solution better than $(1-\epsilon')\ln n$ times the optimum unless there is an $n^{O(\log\log n)}$ time algorithm for NP-complete problems, for any fixed ϵ' with $0 < \epsilon' < 1$, where n is the number of nodes in the network. We finally reduce the problem to a directed Steiner tree problem on an auxiliary directed graph. As a result, any approximation solution for the directed Steiner tree problem gives an approximation solution for the problem with the same degree accuracy, i.e, there is an approximation algorithm for the problem which runs in time $O(k^2 n + km + kn\log(kn) + (kn)^{1/\epsilon}|S|)$, and delivers a solution within $O(|S|^\epsilon)$ the optimum for any fixed ϵ with $0 < \epsilon \leq 1$. Furthermore, a distributed algorithm is also given with $O(km)$ communication complexity and $O(kn)$ time complexity where m is the number of links and k is the number of wavelengths in the network.

The rest of this paper is arranged as follows. In Section 2 we first introduce the network model. We then define the optimal multicast tree problem. In Section 3 we first show the NP-hardness of the problem, then propose a reduction method to reduce the problem to a directed Steiner tree problem on an auxiliary graph. Meanwhile, we also present a distributed algorithm for the problem in this section. We conclude our discussion in Section 4.

2 Network Model and Problem Statement

The optical network is modeled by a directed graph $G = (V, E)$, where V and E are the set of nodes (vertices) and the set of directed links (edges) in the network. Denote by $n = |V|$ and $m = |E|$. Let $d_{in}(G, v)$ and $d_{out}(G, v)$ be the in-degree and the out-degree of v in G. Then, denote $d_{in} = \max\{d_{in}(G, v) \mid v \in V\}$, $d_{out} = \max\{d_{out}(G, v) \mid v \in V\}$, and $d = \max\{d_{in}, d_{out}\}$. It is well known that $\sum_{v\in V} d_{in}(G, v) = \sum_{v\in V} d_{out}(G, v) = m$. Clearly, $m \leq dn$. Suppose that a set $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_k\}$ of wavelengths is available for G. By the definition of Chlamtac et al [3], the *cost structure* of using the resources in the network is represented as follows. For each link e and wavelength λ_i a nonnegative weight $w(e, \lambda_i)$ is given, representing the "cost" of using wavelength λ_i on link e. If λ_i is not available on the link, then the weight is infinite. The "cost" of wavelength conversion is modeled via cost factors of the form $c_v(\lambda_p, \lambda_q)$, which is the cost of wavelength conversion at node v from wavelength λ_p to wavelength λ_q. For some given v, p and q, if the conversion at v is not available, then $c_v(\lambda_p, \lambda_q)$ is infinite. If the two wavelengths are equal (i.e, $p = q$), then $c_v(\lambda_p, \lambda_p) = 0$. The above defined wavelength conversion costs accommodate the general case where conversion cost depends on the nodes and the wavelengths involved.

Let $s \in V$ be a resource, $S \subset V$ be a set of destinations, and every $v \in S$ be reachable from s. Assume that a set $\Lambda(e)$ $(\subseteq \Lambda)$ of available wavelengths associated with each link $e \in E$ has been given, so has the wavelength conversion function at each node $v \in V$. A *multicast tree* rooted at s of G is a tree such that every node in S is reachable from s. Clearly such a tree is a directed tree in which the in-degree of every node is one except the root whose in-degree is zero. Let T be a multicast tree rooted at s including all nodes in S. Assume that $p(v)$ is the parent of v in T if $v \neq s$, otherwise $p(s) = \emptyset$. For convenience, let $w(\langle\emptyset, s\rangle, \lambda) = 0$ for any $\lambda \in \Lambda$. Then the *cost*, $C(T)$, of T is defined as follows.

$$C(T) = \sum_{v\in T} w(\langle p(v), v\rangle, \lambda(v)) + \sum_{v\in T-\{s\}} c_{p(v)}(\lambda(p(v)), \lambda(v))$$

where $\lambda(v)$ represents the wavelength used on link $\langle p(v), v\rangle$ for broadcasting and $c_{p(v)}(\lambda(p(v)), \lambda(v))$ represents the cost of wavelength conversion from $\lambda(p(v))$ to $\lambda(v)$ at node $p(v)$.

The *optimal multicast tree problem* then is to find a multicast tree T in G with minimizing $C(T)$. Note that, for this problem, not only do we need to find such an optimal multicast tree, but also do we need to assign a specific wavelength $\lambda \in \Lambda(\langle p(v), v\rangle)$ for each link $\langle p(v), v\rangle$ and to set the wavelength conversion switches at non-leaf nodes if necessary. Clearly, the optimal semilightpath problem defined by Chlamtac et al [3] is a special case of our problem where $S = \{t\}$. However, as remarked by Chlamtac et al, it is not forbidden in general for a semilightpath to visit a node more than once, on different wavelengths. It is expected, however, that in practical cases the cost structure will exclude such cases from being optimal, but we do not have to exclude them *a priori*. The case where the same link is traversed on the same wavelength more than once is automatically excluded from the potential optimal solutions, since it can be shortened by a shortcut. Therefore, the optimal multicast tree found may be a pseudo-tree, in which it may exist cycles. But in practice by adjusting the "cost structure", this situation can be avoided.

3 The Optimal Multicast Tree Problem

3.1 NP hardness

Before we proceed, we introduce the directed Steiner tree definition. Let $G(V, E)$ be a directed weighted graph with a given subset S of V, a tree of G rooted at a node s including all nodes of S is called *a directed Steiner tree* if the weighted sum of all links in the tree is minimized and every node in S is reachable from s. For this tree we have the following lemma.

Lemma 1 *Given a directed weighted graph $G = (V, E)$, and a set $S \subset V$ of nodes and a resource s, assume that every node in S is reachable from s, finding a directed Steiner tree is NP-complete.*

Proof. The proof is straightforward, omitted. □

Consider a special case of the optimal multicast problem on the network G where the wavelength conversion is not allowed at any node and the wavelength set $\Lambda(e) = \{\lambda\}$ for each link $e \in E$. It is clear that the problem now is the directed Steiner tree problem on G with edge weight $w(e, \lambda)$ for every link $e \in E$. By Lemma 1 the directed Steiner tree problem is NP-complete, so, the optimal multicast problem on G is NP-complete. Therefore, we have

Theorem 1 *Given a directed network $G = (V, E)$ with a set $S \subset V$ of nodes and a resource s and every node in S is reachable from s, assume that a set $\Lambda(e)$ $(\subseteq \Lambda)$ of available wavelengths associated with each link $e \in E$ has been given, so has the wavelength conversion function at each node $v \in V$. Finding an optimal multicast tree on G is NP-complete.*

From Theorem 1, we know that it is unlikely to have an exact solution for the problem in polynomial time unless P=NP. Instead, we shall look for an approximation algorithm for the problem. However, in what follows, we further show that it is unlikely that there is a polynomial approximation algorithm for this problem which delivers a solution better than $(1 - \epsilon')\ln n$ times the optimum unless there is an $n^{O(\log\log n)}$ time algorithm for NP-complete problems, for any fixed ϵ' with $0 < \epsilon' < 1$.

Theorem 2 *Given a directed network $G = (V, E)$ with a set $S \subset V$ of nodes and a resource s and every* *node in S is reachable from s, assume that a set $\Lambda(e)$ ($\subseteq \Lambda$) of available wavelengths associated with each link $e \in E$ has been given, so has the wavelength conversion function at each node $v \in V$. For the optimal multicast tree problem on G, it is unlikely that there exists a polynomial approximation algorithm which delivers a solution better than $(1-\epsilon')\ln n$ times the optimum unless there is an $n^{O(\log\log n)}$ time algorithm for NP-complete problems, for any fixed ϵ' with $0 < \epsilon' < 1$.*

Proof. Consider the above special case of the problem again. In this case the problem becomes a directed Steiner tree problem. It is already known that the minimum set cover problem can be reduced to a directed Steiner tree problem [15]. Therefore, the directed Steiner tree problem is at least as hard as the minimum set cover problem. While Feige [8] has shown that there does exist any polynomial algorithm for the minimum set cover problem which delivers a solution better than $(1 - \epsilon')\ln n$ times the optimum unless there is an $n^{O(\log\log n)}$ time algorithm for NP-complete problems for any fixed ϵ' with $0 < \epsilon' < 1$, where n is the maximum cardinality of the subsets. Therefore, there is no better approximation algorithm for the directed Steiner tree problem which gives a solution better than $(1 - \epsilon')\ln n$ times the optimum unless there is an $n^{O(\log\log n)}$ time algorithm for NP-complete problems, for any fixed ϵ' with $0 < \epsilon' < 1$. Since the optimal multicast tree problem on G is at least as hard as the directed Steiner tree problem, the theorem then follows. □

3.2 Data structures

In [14], Liang et al presented an improved algorithm for finding an optimal semilightpath in WDM networks by introducing new data structures. In what follows, we adopt a variant of their data structures for the optimal multicast tree problem. Given the network $G = (V, E)$, let $E_{in}(G, v)$ and $E_{out}(G, v)$ be the set of incoming links and the set of outgoing links of v in G respectively.

We first construct a directed *multigraph* $G_M = (V_M, E_M)$ as follows. $V_M = V$, and for each directed link $e = \langle u, v\rangle \in E$ there are $|\Lambda(e)|$ parallel directed links in E_M from u to v if $\Lambda(e) \neq \emptyset$. Each such a link is associated with a distinct wavelength $\lambda \in \Lambda(e)$ and weight $w(e, \lambda)$. Thus, G_M has $|V_M| = |V| = n$ nodes and $m_1 = |E_M| = \sum_{e\in E} |\Lambda(e)| \leq km$ links. It is obvious that $|E_{in}(G_M, v)| + |E_{out}(G_M, v)| = \sum_{e\in E_{in}(G,v)\cup E_{out}(G,v)} |\Lambda(e)| \leq k(d_{in}(G, v) + d_{out}(G, v)) \leq 2kd$ for each node $v \in V_M$.

Let $\Lambda_{in}(G_M, v) = \bigcup_{e\in E_{in}(G_M,v)} \Lambda(e)$ and $\Lambda_{out}(G_M, v) = \bigcup_{e\in E_{out}(G_M,v)} \Lambda(e)$ be the set of wavelengths on the incoming links and the set of wavelengths on the outgoing links of v respectively. We then construct a directed bipartite weighted graph $G_v = (X_v, Y_v, E_v, \omega_1)$ for every node $v \in V_M$ as follows. For each wavelength $\lambda \in \Lambda_{in}(G_M, v)$ there is a corresponding node x in X_v, and for each wavelength $\lambda' \in \Lambda_{out}(G_M, v)$ there is a corresponding node y in Y_v. There is a directed link $e = \langle x, y\rangle \in E_v$ from x to y if and only if one of the following conditions holds. (i) $\lambda = \lambda'$. The weight of e is assigned $\omega_1(e) = C_v(\lambda, \lambda) = 0$; (ii) $\lambda \neq \lambda'$ and the wavelength conversion from λ to λ' at v is allowed. The weight of e is assigned $\omega_1(e) = C_v(\lambda, \lambda')$.

Third, we construct an auxiliary graph $G' = (V', E', \omega_2)$ from G_M and G_v for all $v \in V_M$, which is also a directed weighted graph, as follows. $V' = \bigcup_{v\in V_M}(X_v \cup Y_v)$. Let $\langle u, v\rangle \in E_M$ be any link with wavelength λ. Assume that $u' \in Y_u$ and $v' \in X_v$ are the corresponding nodes of u and v in G_u and G_v respectively. Then, $\langle u', v'\rangle \in E'$ and its weight is $\omega_2(\langle u', v'\rangle) = w(\langle u, v\rangle, \lambda)$. Let E_{org} be the set of the links of G' obtained from E_M by the above transformation, then $|E_{org}| = |E_M| = \sum_{e\in E} |\Lambda(e)| \leq km$. For each $e \in \bigcup_{v\in V} E_v$, $\omega_2(e) = \omega_1(e)$. Define $E' = \bigcup_{v\in V_M} E_v \cup E_{org}$.

Lemma 2 *Let $G'(V', E', \omega_2)$ be the directed weighted graph defined as above, then $|V'| = \sum_{v\in V_M}(|X_v| + |Y_v|) = \sum_{v\in V_M}(|\Lambda_{in}(G_M, v)| + |\Lambda_{out}(G_M, v)|) \leq 2kn$ and $|E'| = \bigcup_{v\in V_M} |E_v| + |E_{org}| \leq k^2n + m_1 \leq k^2n + km$.*

Lemma 3 *Let $G'(V', E', \omega_2)$ be the directed weighted graph defined as above. Then G' can be constructed in $O(k^2n + km)$ time and space if G' is represented by adjacency lists.*

We finally construct an auxiliary directed, weighted graph $GC = (VC, EC, \omega_3)$ as follows. $VC = \bigcup_{v\in V}(X_v \cup Y_v) \cup \{v'' \mid v \in V - \{s\}\} \cup \{s'\}$ and $EC = \bigcup_{v\in V} E_v \cup E_{org} \cup \bigcup_{v\in V-\{s\}}\{\langle u, v''\rangle \mid u \in X_v\} \cup \{\langle s', u\rangle \mid u \in Y_s\}$. For every $e \in \bigcup_{v\in V} E_v \cup E_{org}$, $\omega_3(e) = \omega_2(e)$. The links in $\{\langle u, v''\rangle \mid u \in X_v\}$ are assigned weight zeros, i.e, $\omega_3(\langle u, v''\rangle) = 0$, and the links in $\{\langle s', u\rangle \mid u \in Y_s\}$ are assigned weight zeros, i.e, $\omega_3(\langle s', u\rangle) = 0$. It is obvious that GC can be constructed in time $O(k^2n + km)$.

3.3 An approximation algorithm

The basic idea behind our algorithm is to reduce this problem on G to a directed Steiner tree problem on GC defined as above. As a result, any feasible solution for the directed Steiner tree problem on GC corresponds to a feasible solution for the optimal multicast problem on G with the same degree accuracy. We now consider how to find an approximation solution for the problem.

Theorem 3 *Let T be a directed Steiner tree of GC with the source s' and the destination set S' and T_1 be an optimal multicast tree of G, then the cost of T is equal to the cost of T_1, i.e., $\sum_{\langle u,v\rangle\in E(T)} \omega_3(\langle u, v\rangle) = C(T_1)$, where $S' = \{v'' \mid v \in S\}$ and $E(T)$ is the set of links in T.*

Proof. Let $T_1(V_1, E_1)$ be an optimal multicast tree of G rooted at s that includes all nodes in S. We now construct a new graph T_2 from T_1, using the same transformation approach as that for G' from G_M, as follows. Let v be a node of T_1, $c(T_1, v)$ be the set of children of v in T_1, and $p(T_1, v)$ be the parent of v in T_1. Then $|\Lambda_{in}(T_1, v)| = 1$ and $|\Lambda_{out}(T_1, v)| \leq |c(T_1, v)|$. Let $\Lambda_{in}(T_1, v) = \{\lambda(v)\}$, then there is a corresponding node in X_v of G' driven from the link $\langle p(T_1, v), v\rangle$ and the wavelength $\lambda(v)$. We use $\lambda(v)$ to represent the node if there is no any confusion. We also use Z_v to represent the set of corresponding nodes of $\Lambda_{out}(T_1, v)$ in G'. Clearly $Z_v \subseteq Y_v$. Define $V_2 = \bigcup_{v\in T_1}(\{\lambda(v)\} \cup Z_v)$ and $\langle\lambda(v), u\rangle \in E_2$

for all $u \in Z_v$ and the weight of this link is assigned by $c_v(\lambda(p(T_1, v)), \lambda(v))$ if $v \neq s$. For every link $\langle u, v\rangle \in E_1$, $\langle \lambda(u), x_v\rangle \in E_2$, and the weight of the link is assigned with $w(\langle p(T_1, v), v\rangle, \lambda(v))$, where x_v is the corresponding node of v in Z_v. Obviously T_2 is a forest in G' in which each non-root node has in-degree one and every rooted node has in-degree zero. Then we extend T_2 in the following way. For every $v \in T_1$, if $v \neq s$, we add a new node v'' and a new link $\langle \lambda(v), v''\rangle$ to T_2, and assign this link with weight zero. Otherwise, we add a new node s' and new links $\langle s', u\rangle$ to T_2 for all $u \in Z_s$ and assign these new links with weight zeros. Let T_3 be the augmented graph of T_2. Now T_3 is a directed tree of GC which includes all nodes in S'. Let $w(T_1)$ be the cost of the optimal multicast tree of T_1 of G. Then $w(T_2) = w(T_1)$ by the definition. Since the new links added from T_2 to T_3 with weight zeros, $w(T_3) = w(T_2)$. While T is a directed Steiner tree of GC rooted at s' including all nodes of S', $w(T) \leq w(T_3) = w(T_2) = w(T_1)$. Meanwhile, by the definition of T_1, $w(T_1) \leq w(T)$. Therefore, the theorem then follows. □

By Theorem 3, if we can find a directed Steiner tree T of GC, we can give an exact solution for our problem. The transformation from T to the optimal multicast tree of G is straightforward. However, as shown in Lemma 1, finding T of GC is NP-complete. Instead, our objective is to find a polynomial approximation solution for the problem.

Let $G(V, E)$ be a weighted directed graph with a source s and a destination set S ($S \subset V$), the objective is to find a directed approximate Steiner tree. There are several approximation algorithms for this problem [17, 15, 5]. Among them, the most popular and trivial algorithm is based on the shortest path algorithm, which is presented as follows. Finding a shortest path P_v in G from s to v for each $v \in S$. Let $P_{v_1}, P_{v_2}, \ldots, P_{v_{|S|}}$ be the sequence of the paths, sorted by their lengths in increasing order, where $v_i \in S$. Let T be the approximation Steiner tree. $T = P_{v_1}$ and $i = 1$ initially. We repeat the following steps until all paths are added to T: $i = i + 1$, add P_{v_i} to T, and remove those edges from P_{v_i} which introduce a cycle in $T \cup P_{v_i}$. Obviously the solution obtained is within $|S|$ times the optimum because the length of each shortest path is no more than the cost of the directed Steiner tree. In fact there is anther approach to construct T, which is described as follows. Construct a shortest path tree T' rooted at s, then remove those leaves of T' that are not in S until all leaves of T' are in S. The final T' is an approximation Steiner tree. Clearly the time complexity for this latter approach is $O(m + n \log n)$ [6].

Recently Charikar et al [5] presented an efficient approximation algorithm for the directed Steiner tree problem. Their algorithm runs in time $O(\max\{m + n \log n, |S|n^i\})$, and delivers a solution which is $(3i)^i |S|^{1/i}$ optimal where i (≥ 1) is a fixed integer. Note that when $i = 1$, their algorithm takes $O(m + n \log n)$ time. In other words, their algorithm runs in time $O(m + n \log n + n^{1/\epsilon}|S|)$, and delivers a solution which is $O(|S|^\epsilon)$ optimal for any fixed $0 < \epsilon \leq 1$. In summary, we have

Lemma 4 *Let $G(V, E)$ be a weighted directed graph with a source s and a destination set S ($S \subset V$), $|V| = n$ and $|E| = m$, There is a simple algorithm for the directed Steiner tree problem which runs in $O(m + n \log n)$ time and delivers a solution within $|S|$ times the optimum. There is also an efficient algorithm for this problem which runs in time $O(|S|n^{1/\epsilon})$, and delivers a solution which is $O(|S|^\epsilon)$ optimal for any fixed ϵ with $0 < \epsilon < 1$.*

Therefore we have

Theorem 4 *Given a directed network $G = (V, E)$ with a given source s and a destination set S, assume that each link e of G has been assigned with a set $\Lambda(e) \subseteq \Lambda$ of available wavelengths, and every node has been given a wavelength conversion function. There is an approximation algorithm for the optimal multicast tree problem which runs in time $O(k^2 n + km + (kn)^{1/\epsilon}|S|)$, and delivers a solution within $O(|S|^\epsilon)$ the optimum for any fixed ϵ with $0 < \epsilon \leq 1$.*

Proof. The construction of GC and the assignment of links in GC can be done in time $O(k^2 n + km)$ clearly. GC contains $O(kn)$ nodes. Then, finding an approximate Steiner tree T in GC rooted at s' including all nodes in $S' = \{v'' \mid v \in S\}$ by the above simple algorithm and the algorithm due to Charikar et al [5]. T can be found either in time $O(k^2 n + km + kn \log(kn))$, and the cost of T is $|S|$ optimal, or in time $O((kn)^{1/\epsilon}|S|)$, and the cost of T is within $O(|S|^\epsilon)$ the optimal for any fixed $0 < \epsilon < 1$. Therefore, the theorem follows. □

When $S = V - \{s\}$, the optimal multicast tree problem becomes *the optimal broadcast tree problem*. The algorithm suggested in the above is also applicable to this special case. Therefore, we have

Corollary 1 *Given a directed network $G = (V, E)$ with a given source s and a destination set $V - \{s\}$, assume that each link e of G has been assigned with a set $\Lambda(e) \subseteq \Lambda$ of available wavelengths, and every node has been given a wavelength conversion function. There is an approximation algorithm for the optimal broadcast tree problem which runs in time $O(k^2 n + km + kn \log(kn) + k^{1/\epsilon} n^{1+1/\epsilon})$, and delivers a solution within $O(n^\epsilon)$ the optimal for any fixed $0 < \epsilon \leq 1$.*

Proof. Set $S = V - \{s\}$, and then apply Theorem 4, the corollary follows. □

3.4 A distributed algorithm

Usually the optical network can be decomposed into two separate networks in functionality. One is called *data network* which is used to transfer large volume data such as image data and databases etc, by its optical fibers. The other one is called *control network* which is used to for high level protocol controls such as finding an approximate optimal multicast tree and setting appropriate optical switches for every node in the tree. Because the size of each message used for control is usually not large, people make use of the electronic network to implement high level protocol controls. Since we are dealing with a large wide area network, in practice it is not realistic to design a centralized algorithm to find such an approximate optimal multicast tree for the network. Instead, a distributed algorithm seems more appropriate. In the following we suggest a distributed algorithm for the problem. The approach is to embed the ideal network GC into the physical network G first. Then use G to simulate GC. In doing so we show that the construction of GC has high locality, which is explained as follows. In the original network $G(V,E)$, we construct a bipartite weighted graph $G'_v(X'_v, Y'_v, E'_v, \omega_3)$ for every node $v \in V$ as follows. Let us recall the construction of $G_v(X_v, Y_v, E_v, \omega_1)$ for every $v \in V_M = V$. Now we construct another bipartite graph $G'_v = (X'_v, Y'_v, E'_v, \omega_3)$. If $v \neq s$, then $X'_v = X_v$, $Y'_v = Y_v \cup \{v''\}$, and $E'_v = E_v \cup \{\langle u, v''\rangle \mid u \in X_v\}$, all links in $E'_v - E_v$ are assigned weight zeros, i.e, $\omega_3(e) = 0$ for every $e \in E'_v - E_v$; and for every $e \in E_v$, $\omega_3(e) = \omega_1(e)$. Otherwise $X'_s = X_s \cup \{s'\}$, $Y'_s = Y_s$, $E'_s = E_s \cup \{\langle s'', u\rangle \mid u \in Y_s\}$ and for every $e \in E'_s - E_s$, $\omega_3(e) = 0$. For every $e \in E_s$, $\omega_3(e) = \omega_1(e)$. Then the auxiliary directed weighted graph $GC = (VC, EC)$ can be constructed as follows. $VC = \bigcup_{v \in V}(X'_v \cup Y'_v)$ and $EC = \bigcup_{v \in V} E'_v \cup E_{org}$. Note that the links in E_{org} are simulated by the physical links in E of G. Each physical link $e \in E$ of G serves as $|\Lambda(e)|$ links of GC. As a result, GC is constructed and presented distributively.

The optimal multicast tree problem on G then becomes finding a directed Steiner tree problem on GC with a given source s' and a set $S' = \{v'' \mid v \in S\}$ of destinations. While the Steiner tree problem on undirected graphs is a well studied problem in the distributed computing environment, there are many efficient algorithms for it such as the algorithm by Kompella et al. [13]. Here we look for a distributed algorithm for the directed Steiner tree problem. Those algorithms for undirected graphs may not be appropriate for our case. In our case, we make use of the distributed algorithm for finding a shortest path between a pair of nodes by Chandy and Misra [2], which runs in time $O(n')$ using $O(m')$ messages in a network H with n' nodes and m' links. Actually the result delivered by their algorithm is a shortest tree rooted at the source. Therefore we have

Theorem 5 *Given a directed network $G = (V,E)$ with a given source s and a destination set S, assume that each link e of G has been assigned with a set $\Lambda(e) \subseteq \Lambda$ of available wavelengths, and every node has been given a wavelength conversion function. There is a distributed, approximation algorithm for finding the optimal multicast tree on G. The communication and time complexities of the algorithm are $O(km)$ and $O(kn)$ respectively, and the solution delivered is $|S|$ times the optimum.*

Proof. First we construct the weighted directed graph GC which takes $O(1)$ time because the local computation is negligible in the distributed computing model. Then we make use of the algorithm of Chandy and Misra [2] to find a shortest path tree T in GC rooted at s'. This step requires $O(kn)$ time and $O(km)$ messages since GC contains $O(kn)$ nodes. Note that the links in $\bigcup_{v \in V} E_v$ are not accounted because they are inside of nodes, so, we reduce $O(k^2 n)$ messages in this step. Finally we remove all leaves that are not in $S' = \{v'' \mid v \in S\}$ from T until all its leaves are in S'. This step requires $O(kn)$ time and $O(kn)$ messages because T is a tree. Clearly the solution obtained is $|S|$ times the optimum. $\Box$

4 Conclusion

In this paper we first formalized the optimal multicast tree problem in wide area WDM networks in terms of the cost of using wavelengths on links and the cost for doing wavelength conversion at nodes if necessary. We then showed the NP hardness of the problem. We finally reduced the problem to a directed Steiner tree problem on an auxiliary directed weighted graph. As a result, the approximation solution for the directed Steiner tree problem gives an approximation solution for our problem with the same degree accuracy.

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